

Relating \hat{q} , η/s and ΔE in an expanding Quark-Gluon Plasma

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We use linear viscous hydrodynamics to describe the energy and momentum deposited by a fast moving parton in a quark gluon plasma. This energy-momentum is in turn used to compute the probability density for the production of soft partons by means of the Cooper-Frye formula. We use this probability density to render manifest a relation between the average transverse momentum given to the fast moving parton from the medium \hat{q} , the entropy density to shear viscosity ratio η/s and the energy lost by the fast moving parton ΔE in an expanding medium under similar conditions to those generated in nucleus-nucleus collisions at the LHC. We find that \hat{q} increases linearly with ΔE for both trigger and away side partons that have been produced throughout the medium. On the other hand, η/s is more stable with ΔE . We also study how these transport coefficients vary with the geometrical location of the hard scattering that produces the fast moving partons. The behavior of \hat{q} , with ΔE is understood as arising from the length of medium the parton traverses from the point where it is produced. However, since η/s is proportional to the ratio of the length of medium traversed by the fast parton and the average number of scatterings it experiences, it has a milder dependence on the energy it loses. This study represents a tool to obtain a direct connection between transport coefficients and the description of in-medium energy loss within a linear viscous hydrodynamical evolution of the bulk.

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I. INTRODUCTION

The results from experiments where heavy nuclei are collided at high energies, carried both at the BNL Relativistic Heavy-Ion Collider [1] and the CERN Large Hadron Collider [2], show that a state of matter, where quarks and gluons are not confined to individual nucleons is formed, the so called Quark-Gluon Plasma (QGP). The behavior of the QGP's soft bulk, that is particles with $p_T \lesssim 2$ GeV, can be accurately described using viscous hydrodynamics in the liquid regime [3, 4]. One way to study the properties of the QGP is to consider the way that fast moving partons, either quarks or gluons, traveling through the QGP, transfer energy and momentum to the bulk. This energy-momentum is then converted into particles upon hadronization. The process is well described by hydrodynamics, where the source of energy and momentum is the fast moving parton [5–7]. In a gradient expansion to lowest non-trivial order, one can include the viscous effects to first order in the shear viscosity to entropy ratio η/s . This procedure is known as linear viscous hydrodynamics. Extracting the value of this transport coefficient for the QGP is at the core of current efforts both in the theoretical and experimental fronts [8].

An important ingredient for this description is the en-

ergy loss per unit length dE/dx which enters as the coefficient describing the strength of the local hydrodynamic source term. Hadronization of the energy and momentum deposited into the medium can be carried out by means of the Cooper-Frye formula. In this way, linear viscous hydrodynamics provides a means to compute the probability density describing the production of soft-particles from the energy lost by a fast moving parton in the medium [9].

Another important quantity that describes the interaction of fast moving partons and the QGP is the average transverse momentum squared per unit length transferred to the parton from the medium, the so called \hat{q} -parameter. This parameter represents the average broadening of the original fast parton's trajectory resulting from interactions induced by the medium. It has been suggested [10] that \hat{q} for a thermal parton can be related to the shear viscosity to entropy ratio η/s of the QGP plasma, since it is argued that \hat{q} is a measure of the coupling strength of the medium. The relation suggested between these transport coefficients involves the temperature of the medium depending on the coupling strength regime, as

$$\frac{T^3}{\hat{q}} \begin{cases} \approx \frac{\eta}{s} & , \text{ weakly-coupled} \\ \ll \frac{\eta}{s} & , \text{ strongly-coupled.} \end{cases} \quad (1)$$

Furthermore, there are collaborative efforts in the theoretical community to extract the jet transport parameter \hat{q} using different approaches to energy-loss mechanisms for jet quenching at RHIC and LHC energies [11]. Further recent phenomenological studies in connection with underlying anomalies [12] and thermalization of minijets [13] together with recent theoretical proposals using effective field theory to describe transport coefficients [14], make evident the need for elucidating the interplay between these transport coefficients across the phenomenological landscape of the QGP.

Since both η/s and \hat{q} are transport coefficients describing the exchange of energy and momentum between fast partons and medium, a pertinent question is whether there is a quantitative relation between these parameters that can be extracted from the hydrodynamical picture. In this work we provide such relation using linear viscous hydrodynamics and the Cooper-Frye formula to describe the probability to produce soft partons from the energy-momentum deposited by the fast partons to the bulk. We show that a non-trivial relation exists for the case when the medium is considered as non-static. In doing so we also study the scaling of such transport coefficients with the parton's travelled length and their total energy lost while traversing the QGP. The paper is organized as follows: In Sec. II we derive a relation between the transport coefficients \hat{q} and η/s for a static medium. We show that in this case \hat{q} is independent of η/s . In Sec. III we relax the condition of studying a static medium introducing a model for the energy loss that accounts for expansion. We show that under these conditions a non-trivial dependence between these transport coefficients emerges. We also study the effects that the position of the hard scattering and therefore of the path length travelled by the fast moving partons within the plasma, have on the extracted values for the transport coefficients. We show that these coefficients depend on the total energy lost and thus that triggering on events with a certain energy loss will yield different values of these parameters. We finally summarize and conclude in Sec. IV.

II. \hat{q} AND η/s IN A STATIC MEDIUM

The particle's multiplicity is given by the Cooper-Frye formula [15]

$$E \frac{dN}{d^3p} = \frac{1}{(2\pi)^3} \int d\Sigma_\mu p^\mu [f(p \cdot u) - f(p_0)], \quad (2)$$

where $f(p \cdot u) - f(p_0)$ is the phase-space disturbance produced by the fast moving parton on top of the equilibrium distribution $f(p_0)$, with Σ_μ and p_μ representing the freeze-out hypersurface and the particle's momentum, respectively. The medium's total four-velocity $u^\mu \equiv u_0^\mu + \delta u^\mu$ is made out of two parts: the background four-velocity u_0^μ and the disturbance δu^μ . This last contribution is produced by the fast moving parton and can be computed using linear viscous hydrodynamics once

the source, representing the parton, is specified. For a static background (which we assume for the moment) and in the linear approximation, u^μ can be written as

$$u^\mu \equiv u_0^\mu + \delta u^\mu \quad (3)$$

$$= \left(1, \frac{\mathbf{g}}{\epsilon_0(1 + c_s^2)} \right), \quad (4)$$

where the spatial part of the medium's four-velocity, $\mathbf{u} = \mathbf{g}/\epsilon_0(1 + c_s^2)$, is written for convenience in terms of the momentum density \mathbf{g} associated to the disturbance, with ϵ_0 and c_s the static background's energy density and sound velocity, respectively. We focus on events at central rapidity, $y \simeq 0$, and take the direction of motion of the fast parton to be the \hat{z} axis and the beam axis to be the \hat{x} axis. With this geometry, the transverse plane is the $\hat{y} - \hat{z}$ plane and therefore, the momentum four-vector for a (massless) particle is explicitly given by

$$p_\mu = (E, p_x, p_y, p_z) \quad (5)$$

$$= (p_T, 0, p_T \sin \phi, p_T \cos \phi), \quad (6)$$

where ϕ is the angle that the momentum vector \mathbf{p} makes with the \hat{z} axis. We use Bjorken's geometry thus,

$$\begin{aligned} d^3p &= p_T dp_T d\phi dp_x \\ p_x &= p_T \sinh y \\ dp_x &= p_T \cosh y dy \\ E &= p_T \cosh y \end{aligned} \quad (7)$$

and therefore

$$E \frac{dN}{d^3p} = \frac{dN}{p_T dp_T d\phi dy}. \quad (8)$$

For simplicity we consider a freeze-out hypersurface of constant time,

$$d\Sigma_\mu = (d^3r, 0, 0, 0). \quad (9)$$

Therefore, using Eqs. (2) and (8), the particle momentum distribution around the direction of motion of a fast moving parton is given by

$$\frac{dN}{p_T dp_T d\phi d^2r} = \frac{\Delta\tau(\Delta y)^2}{(2\pi)^3} p_T [f(p \cdot u) - f(p_0)], \quad (10)$$

with $\Delta\tau$ the freeze-out time interval, d^2r the surface element in the transverse plane and where we have assumed a perfect correlation between the space-time rapidity η and y to substitute $\Delta\eta$ by Δy . We assume that the equilibrium distribution is of the Boltzmann type. Assuming that the energy density and temperature are related through Boltzmann's law

$$\epsilon \propto T^4, \quad (11)$$

one gets

$$\frac{\delta T}{T_0} = \frac{\delta \epsilon}{4\epsilon_0}, \quad (12)$$

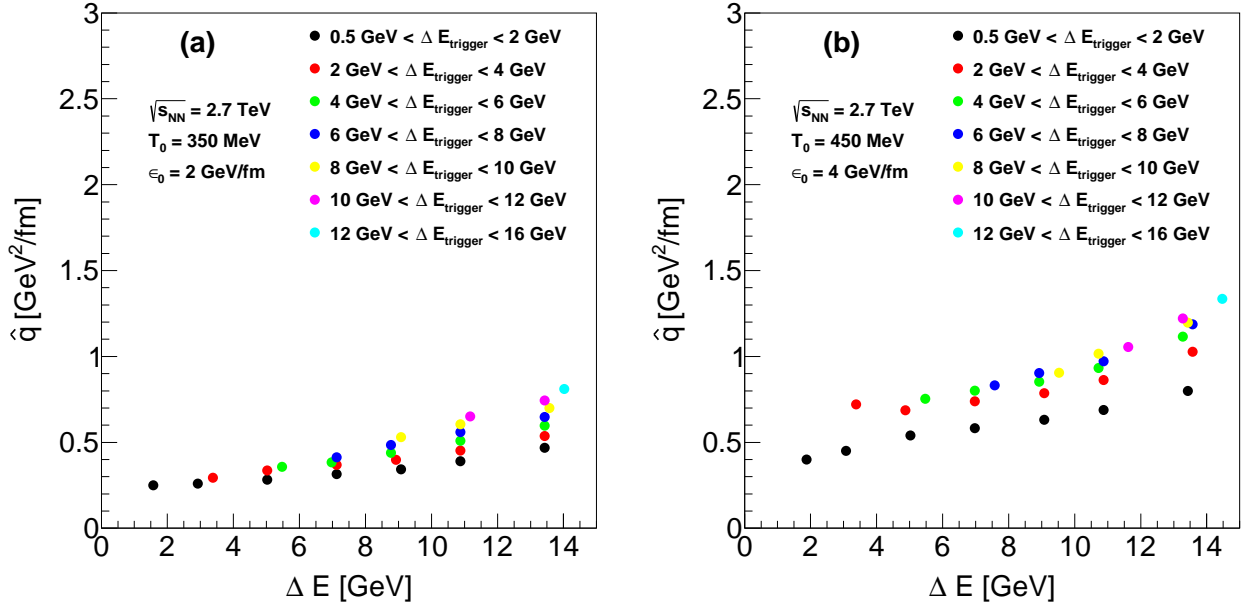


FIG. 1. \hat{q} as a function of ΔE for the away-side particle for different trigger particle energy losses. The left (right) panel corresponds to a QGP temperature $T_0 = 350$ ($T_0 = 450$) MeV and a corresponding $\epsilon_0 = 2$ ($\epsilon_0 = 4$) GeV/fm in the $1-d$ energy loss model.

where T_0 is the background medium's temperature and δT is the change in temperature caused by the passing of the fast parton.

Since for the validity of linearized hydrodynamics, both $\delta\epsilon$ and \mathbf{g} need to be small quantities compared to ϵ_0 , we can expand the difference $f(p \cdot u) - f(p_0)$ to linear order

$$f(p \cdot u) - f(p_0) \simeq \left(\frac{p_T}{T_0} \right) \exp[-p_T/T_0] \times \left(\frac{\delta\epsilon}{4\epsilon_0} + \frac{\mathbf{g}_y \sin \phi + \mathbf{g}_z \cos \phi}{\epsilon_0(1 + c_s^2)} \right). \quad (13)$$

The energy and momentum densities, $\delta\epsilon$ and \mathbf{g}_i , can be found as the solution of the linearized viscous hydrodynamical equations once the source representing the fast moving parton is specified. The current associated to the source is given by

$$J^\nu(\mathbf{r}, t) = \left(\frac{dE}{dx} \right) v^\nu \delta^3(\mathbf{x} - \mathbf{v}t), \quad (14)$$

where (dE/dx) is the energy loss per unit length. $\delta\epsilon$ and \mathbf{g}_i are given then by

$$\delta\epsilon = \left(\frac{1}{4\pi} \right) \left(\frac{dE}{dx} \right) \left(\frac{2v}{3\Gamma_s} \right)^2 \left(\frac{9}{8v} \right) I_{\delta\epsilon}(\alpha, \beta) \quad (15)$$

and

$$\mathbf{g}_i = \left(\frac{1}{4\pi} \right) \left(\frac{dE}{dx} \right) \left(\frac{2v}{3\Gamma_s} \right)^2 I_{\mathbf{g}_i}(\alpha, \beta), \quad (16)$$

where the integrals $I_{\delta\epsilon}$ and $I_{\mathbf{g}_i}$ are dimensionless functions representing the collected energy-momentum deposited by the source term when moving through the medium and are given in Refs. [16] in terms of dimensionless variables α and β which encode the distance to the source, in units of the sound attenuation length

$$\Gamma_s \equiv \frac{4\eta}{3\epsilon_0(1 + c_s^2)}. \quad (17)$$

Therefore, the particle momentum distribution around the direction of motion of a fast moving parton is given, in the linear approximation, by

$$\frac{dN}{p_T dp_T d\phi d^2r} = \frac{\Delta\tau(\Delta y)^2}{(2\pi)^3} \frac{p_T^2}{T_0} \exp[-p_T/T_0] \times \left(\frac{\delta\epsilon}{4\epsilon_0} + \frac{\mathbf{g}_y \sin \phi + \mathbf{g}_z \cos \phi}{\epsilon_0(1 + c_s^2)} \right). \quad (18)$$

The probability density $\mathcal{P}(p_T, r, \phi)$ is obtained from the above equation dividing by the total number of particles produced, N , which in turn is obtained integrating

over all phase space. Therefore

$$\mathcal{P}(p_T, r, \phi) = \frac{1}{N} \frac{dN}{p_T dp_T d^2r}. \quad (19)$$

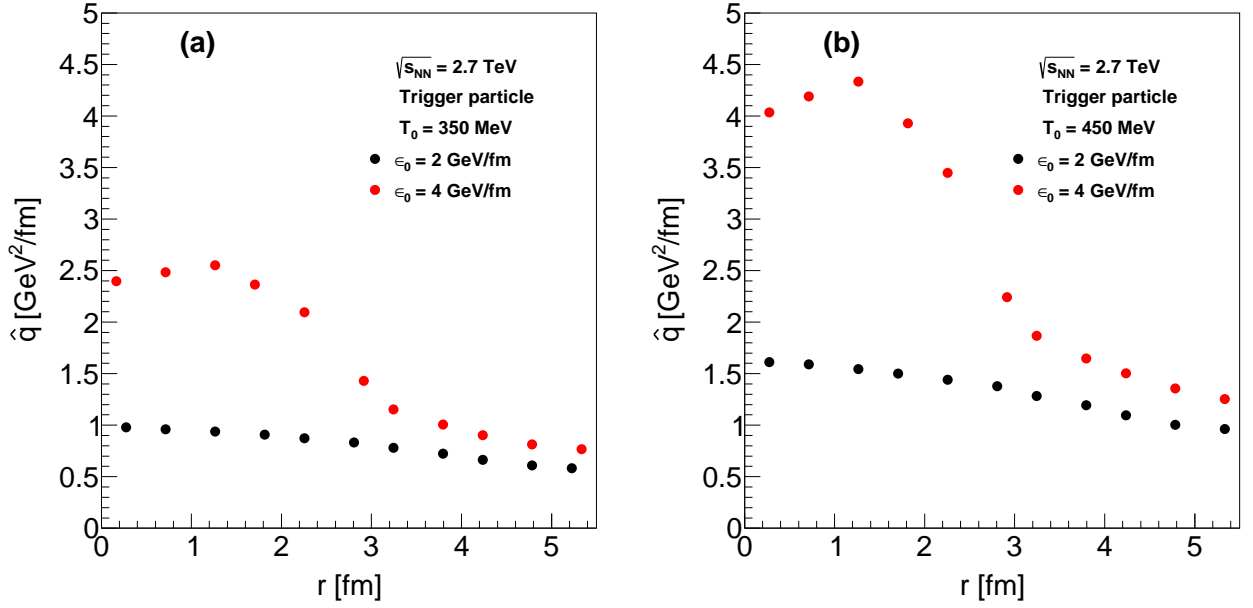


FIG. 2. \hat{q} for the trigger particle as a function of the distance r where the hard scattering took place. The left (right) panel corresponds to a QGP temperature $T_0 = 350$ ($T_0 = 450$) MeV for the two values of $\epsilon_0 = 2, 4$ GeV/fm in the $1-d$ energy loss model.

Note that the average momentum squared carried by the disturbance, transverse to the direction of the fast parton's motion is given by

$$\langle q^2 \rangle \equiv \int d^2r \int dp_T p_T \times 2 \int_0^{\pi/2} d\phi \mathcal{P}(p_T, r, \phi) p_T^2 \sin^2 \phi, \quad (20)$$

where in the integration over the angle between the fast parton and the produced particles we have implemented the condition to consider that these last move in the forward direction, namely, the direction of the fast parton.

Using Eqs. (18) and (19), $\langle q^2 \rangle$ can be explicitly written as

$$\langle q^2 \rangle = \frac{1}{N} \frac{\Delta\tau(\Delta y)^2}{(2\pi)^3} \frac{120T_0^5}{\epsilon_0} \times \int d^2r \left[\frac{\pi}{8} \delta\epsilon + \frac{(4/3)\mathbf{g}_y + (2/3)\mathbf{g}_z}{(1+c_s^2)} \right]. \quad (21)$$

Since N is in turn given by

$$N = \int d^2r \int dp_T p_T \times 2 \int_0^{\pi/2} d\phi \mathcal{P}(p_T, r, \phi) = \frac{\Delta\tau(\Delta y)^2}{(2\pi)^3} \frac{6T_0^3}{\epsilon_0} \int d^2r \left[\frac{\pi}{4} \delta\epsilon + \frac{2\mathbf{g}_y + 2\mathbf{g}_z}{(1+c_s^2)} \right], \quad (22)$$

we finally get

$$\langle q^2 \rangle = 20 T_0^2 \frac{\int d^2r \left[\frac{\pi}{8} \delta\epsilon + \frac{(4/3)\mathbf{g}_y + (2/3)\mathbf{g}_z}{(1+c_s^2)} \right]}{\int d^2r \left[\frac{\pi}{4} \delta\epsilon + \frac{2\mathbf{g}_y + 2\mathbf{g}_z}{(1+c_s^2)} \right]}. \quad (23)$$

Can $\langle q^2 \rangle$ be identified with the average momentum squared *given to the fast parton* by the medium and therefore with \hat{q} upon dividing by the medium's length? The question is pertinent in the sense that the above calculation refers to the average momentum squared *given to the medium by the fast parton*. If the parton's change in energy is small the main effect on the fast parton is a deflection of its original trajectory. This deflection comes along with energy and momentum deposited within the medium via radiation or collisional processes. As a result of energy and momentum conservation during these processes, the momentum put into the medium should compensate the momentum given to the fast parton. In other words, one could expect that the collected overall momentum (squared) should correspond to the equivalent quantity gained by fast parton in the transverse direction. Therefore, since in a hydrodynamical picture, the energy-momentum is described in terms of $\delta\epsilon$ and \mathbf{g} , we can write for the parameter \hat{q}

$$\hat{q} = \frac{\langle q^2 \rangle}{L}, \quad (24)$$

where $\langle q^2 \rangle$ is given by Eq. (23).

Note that for the static case thus discussed, the expression for \hat{q} is essentially independent of η/s . This happens

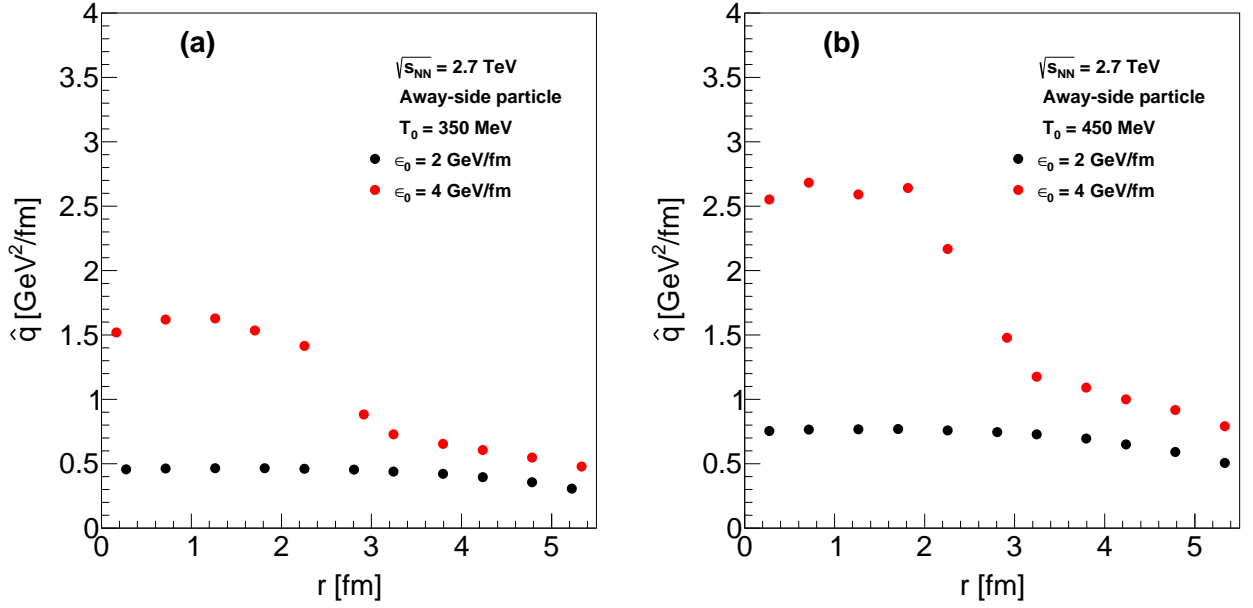


FIG. 3. \hat{q} for the away-side particle as a function of the distance r where the hard scattering took place. The left (right) panel corresponds to a QGP temperature $T_0 = 350$ ($T_0 = 450$) MeV and a corresponding $\epsilon_0 = 2$ ($\epsilon_0 = 4$) GeV/fm in the 1- d energy loss model.

because given the explicit factorization of Γ_s and thus of η/s in Eqs. (15) and (16), the remaining dependence of η/s cancels between numerator and denominator in Eq. (23) since the numerical coefficients accompanying $\delta\epsilon$ and the components of \mathbf{g} are practically the same.

Since the assumption of an static medium is not entirely realistic, we now proceed to study whether an expanding medium makes \hat{q} to depend on η/s .

III. \hat{q} AND η/s IN AN EXPANDING MEDIUM

A full-fledged hydrodynamical computation of \hat{q} in an expanding medium requires a numerical treatment. Let us instead attempt a phenomenological description based

on modelling the way the medium gets diluted during the first stages of the collision due to longitudinal expansion [18]. A fast moving parton loses energy depending on the evolving gluon density that it traverses along its path through the medium, such that

$$\Delta E = \left\langle \frac{dE}{dx} \right\rangle_{1d} \int_{\tau_0}^{\infty} d\tau \frac{\tau - \tau_0}{\tau_0 \rho_0} \rho_g(\tau, \mathbf{b}, \mathbf{r} + \hat{\mathbf{n}}\tau), \quad (25)$$

where the gluon density ρ_g is related to the nuclear geometry of the produced medium as

$$\rho_g(\tau, \mathbf{b}, \mathbf{r}, \hat{\mathbf{n}}) = \frac{\tau_0 \rho_0}{\tau} \frac{\pi R_A^2}{2A} \times [T_A(|\mathbf{r} + \hat{\mathbf{n}}\tau|) + T_A(|\mathbf{b} - \mathbf{r} - \hat{\mathbf{n}}\tau|)]. \quad (26)$$

Here ρ_0 is the central gluon density, T_A the nuclear thickness function, R_A the nuclear radius and A the atomic number, \mathbf{b} the impact parameter of the collision, \mathbf{r} the transverse plane location of the hard scattering where the partons are produced and $\hat{\mathbf{n}}$ the direction in which the produced hard parton travels in the medium. The average number of scatterings $\langle n \rangle$ is given in the model by

$$\langle n \rangle = \int_{\tau_0}^{\infty} d\tau \frac{1}{\lambda_0 \rho_0} \rho_g(\tau, \mathbf{b}, \mathbf{r}, \hat{\mathbf{n}}\tau), \quad (27)$$

where λ_0 is the parton mean free path for a constant density ρ_0 . Since we want to consider the most central

collisions, hereafter we set $\mathbf{b} = 0$. The one dimensional energy loss $\langle dE/dx \rangle_{1d}$ is parametrized as

$$\left\langle \frac{dE}{dx} \right\rangle_{1d} = \epsilon_0 \left[\frac{E}{\mu_0} - 1.6 \right]^{1.2} \left[7.5 + \frac{E}{\mu_0} \right]^{-1}, \quad (28)$$

where E is the energy of the fast moving parton. The mean free path for a constant density ρ_0 is taken as $\lambda_0 = 0.25$ fm. The parameter ϵ_0 is related to λ_0 by $\epsilon_0 \lambda_0 = 0.5$ GeV. We work with a value $\mu_0 = 1.5$ GeV. These parameters are tuned to describe LHC data on R_{AA} [19].

In order to incorporate an expanding medium into the hydrodynamical description of the computation of \hat{q} we approximate the average energy loss per unit length that

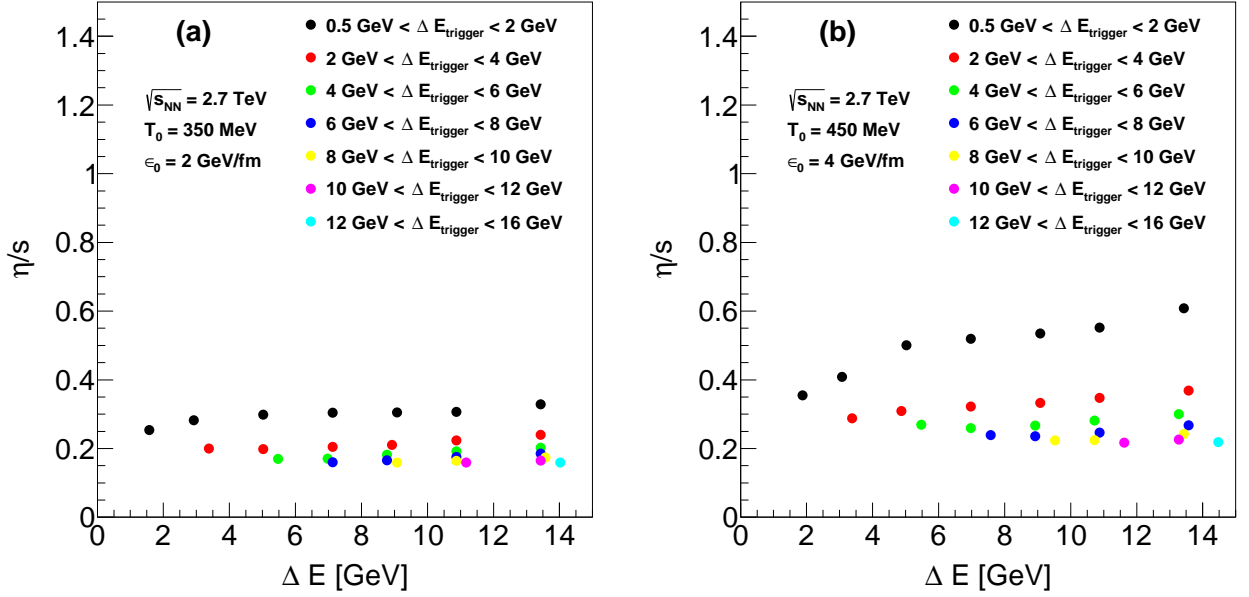


FIG. 4. η/s as a function of ΔE for the away-side particle for different trigger particle energy losses. The left (right) panel corresponds to a QGP temperature $T_0 = 350$ ($T_0 = 450$) MeV and a corresponding $\epsilon_0 = 2$ ($\epsilon_0 = 4$) GeV/fm in the $1-d$ energy loss model.

appears in Eqs. (15) and (16) with the energy loss given by the above described model divided by the in-medium length L travelled by the fast moving parton, namely

$$\left(\frac{dE}{dx}\right) = \frac{\Delta E}{L(\mathbf{r}, \hat{\mathbf{n}})}, \quad (29)$$

where ΔE is given by Eq. (25). Note that with this choice the current $J^\nu(\mathbf{x}, t)$ in Eq. (14) is still constant in space-time, however its amplitude depends on the parton's energy and on the matter density in the expanding

medium. We take for $L(\mathbf{r}, \hat{\mathbf{n}})$ the geometrical distance from the point where the hard scattering took place to the sharp edge of the interaction region, namely

$$L(\mathbf{r}, \hat{\mathbf{n}}) = \frac{1}{2} \left(\sqrt{R_A^2 - r^2 \sin^2 \varphi} - r \cos \varphi \right), \quad (30)$$

where $r = |\mathbf{r}|$ and φ is the angle between \mathbf{r} and $\hat{\mathbf{n}}$. With this choice we account for the fact that within a diluting medium the mean free path $L(\mathbf{r}, \hat{\mathbf{n}})/\langle n \rangle$ becomes larger than in the static case.

After these changes, Eqs. (15) and (16) become

$$\delta\epsilon = \left(\frac{1}{4\pi}\right) \left(\frac{\Delta E}{L(\mathbf{r}, \hat{\mathbf{n}})}\right) \left(\frac{2v}{3\Gamma_s}\right)^2 \left(\frac{9}{8v}\right) I_{\delta\epsilon}(\alpha, \beta) \quad (31)$$

and

$$\mathbf{g}_i = \left(\frac{1}{4\pi}\right) \left(\frac{\Delta E}{L(\mathbf{r}, \hat{\mathbf{n}})}\right) \left(\frac{2v}{3\Gamma_s}\right)^2 I_{\mathbf{g}_i}(\alpha, \beta), \quad (32)$$

To compute \hat{q} we now generate a sample of parton events at random positions \mathbf{r} , moving in random directions $\hat{\mathbf{n}}$ within the medium. This sample is obtained using MadGraph 5 [20] for $2 \rightarrow 2$ parton events in $p + p$ collisions at $\sqrt{s_{NN}} = 2.7$ TeV that subsequently loose energy according to the model thus described. In this manner we

produce a distribution of events characterized by values of ΔE , $L(\mathbf{r}, \hat{\mathbf{n}})$ and $\langle n \rangle$. Therefore \hat{q} can be obtained for instance as a function of ΔE by classifying events with a given amount of energy lost, regardless of where the hard scattering took place or the direction of motion of the fast parton, as

$$\hat{q}_{\Delta E} = \frac{20 T_0^2 \int d^2r \left(\frac{\pi}{8} \delta\epsilon + \frac{(4/3)\mathbf{g}_y + (2/3)\mathbf{g}_z}{(1+c_s^2)} \right)_{\Delta E}}{\sum_{\Delta E} L(\mathbf{r}, \hat{\mathbf{n}}) \int d^2r \left(\frac{\pi}{4} \delta\epsilon + \frac{2\mathbf{g}_y + 2\mathbf{g}_z}{(1+c_s^2)} \right)_{\Delta E}}. \quad (33)$$

Alternatively, \hat{q} can also be obtained as a function of the location of the hard scattering, regardless of the direction the parton travelled or the amount of energy lost.

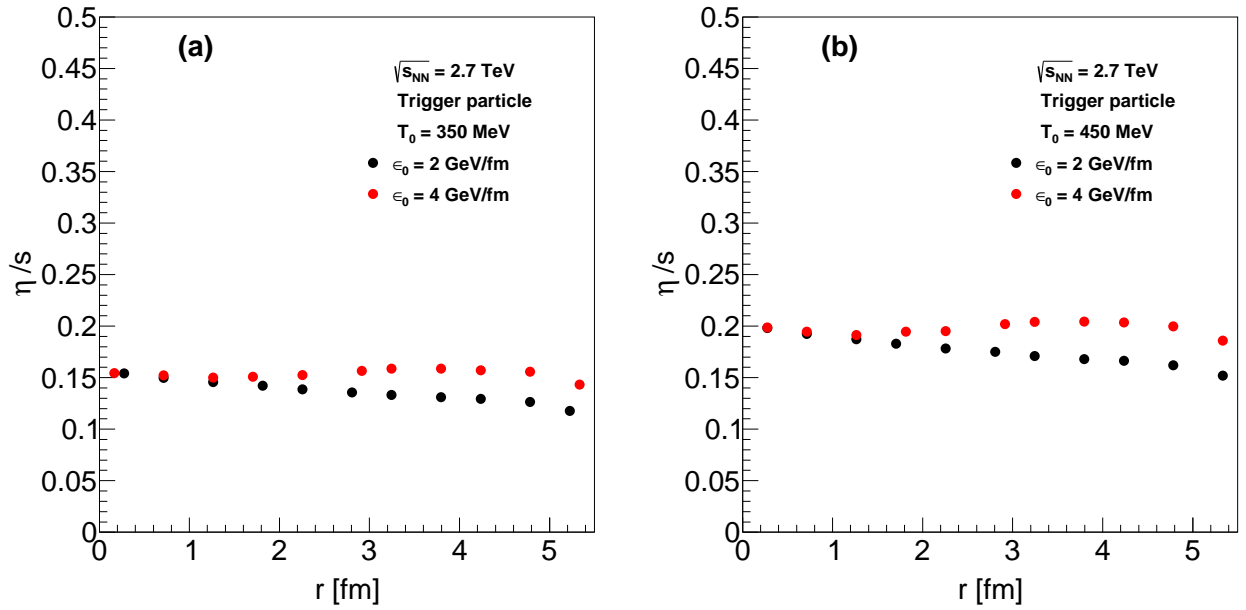


FIG. 5. η/s for the trigger particle as a function of the distance r where the hard scattering took place. The left (right) panel corresponds to a QGP temperature $T_0 = 350$ ($T_0 = 450$) MeV and a corresponding $\epsilon_0 = 2$ ($\epsilon_0 = 4$) GeV/fm in the $1-d$ energy loss model.

Notice that the model can also be used to estimate η/s since we know that this transport coefficient is proportional to the ratio of the medium's mean free path to the thermal wavelength [4, 21]. The model gives, event by event

$$\frac{\eta}{s} \sim T \frac{L(\mathbf{r}, \hat{\mathbf{n}})}{\langle n \rangle}, \quad (34)$$

where $\langle n \rangle$ is given by Eq. (27). Again, we can classify events with a given energy loss, also regardless of where the hard scattering took place or the direction of motion of the fast parton or as a function of the location of the hard scattering, regardless of the direction the parton travelled or the amount of energy lost. In this way we can correlate events with a total amount of energy loss to the corresponding value of η/s which characterizes the amount of medium traversed by the fast parton.

Since the total energy lost by the fast parton bears a relation with the amount of medium the parton traversed, we can also study the transport coefficients classifying the events in terms of the value of r where they were produced. On general grounds one expects that, modulo the influence of the expanding medium, partons that travel a larger path length lose more energy.

Figure 1 shows \hat{q} as a function of the energy lost by the away-side particle for the cases where the trigger particle loses the indicated amount of energy. Hereby, for this and the rest of the figures, the left (right) panel corresponds to a QGP temperature $T_0 = 350$ ($T_0 = 450$) MeV computed with $\epsilon_0 = 2, 4$ GeV/fm in the $1-d$ energy loss model. Notice that the \hat{q} values are widespread and

have a strong dependence on the amount of energy lost both by the trigger and the away-side particles, making it difficult to assign a unique value of this parameter to characterize the plasma.

To study a possible geometrical effect caused by the amount of effective medium travelled by the partons on the value of \hat{q} , Figs. 2 and 3 show \hat{q} for the trigger and associate particles, respectively, as a function of the distance r where the hard scattering took place measured from the center of the interaction region. Notice that \hat{q} has a similar behaviour for trigger and associate particles. There is a strong dependence of the value of \hat{q} on r . The largest value occurs at a distance that maximizes the medium density as this last expands. Notice that contrary to the case when $\epsilon_0 = 2$ GeV/fm, the case $\epsilon_0 = 4$ GeV/fm shows a pronounced maximum for \hat{q} around $r = 1.4$ fm. This behavior can be understood by realizing that for the considered energy in the collision $\sqrt{s_{NN}} = 2.7$ TeV, the average transverse momentum of the produced partons in the hard scattering is of order 15 GeV. For ϵ_0 of order 1-2 GeV/fm partons lose on average 3-5 GeV which is small compared to the 15 GeV that partons were produced with on average. Therefore, since 1-2 GeV is not the parton's most likely value, the behavior of \hat{q} is pretty much flat as a function of r , which is tantamount to ΔE given that partons lose more energy the more medium they travel. On the other hand, for ϵ_0 of order 4 GeV/fm, partons lose on average 10 GeV which is a quantity comparable to the average momentum they were produced with. Since \hat{q} is basically the average mo-

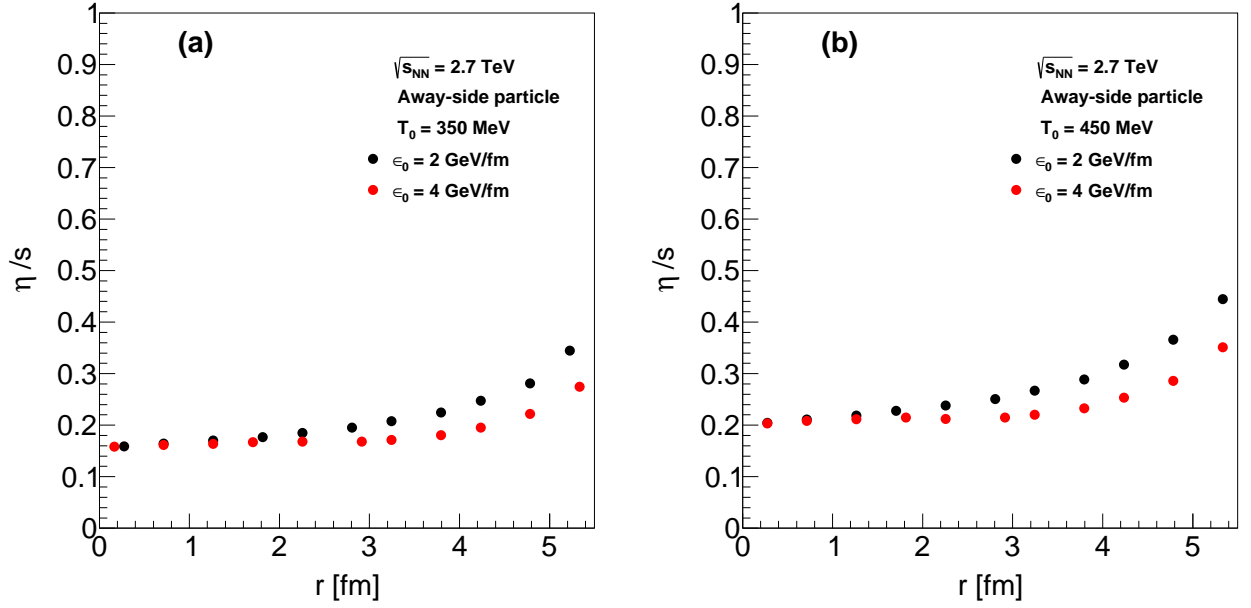


FIG. 6. η/s for the away-side particle as a function of the distance r where the hard scattering took place. The left (right) panel corresponds to a QGP temperature $T_0 = 350$ ($T_0 = 450$) MeV and a corresponding $\epsilon_0 = 2$ ($\epsilon_0 = 4$) GeV/fm in the $1 - d$ energy loss model.

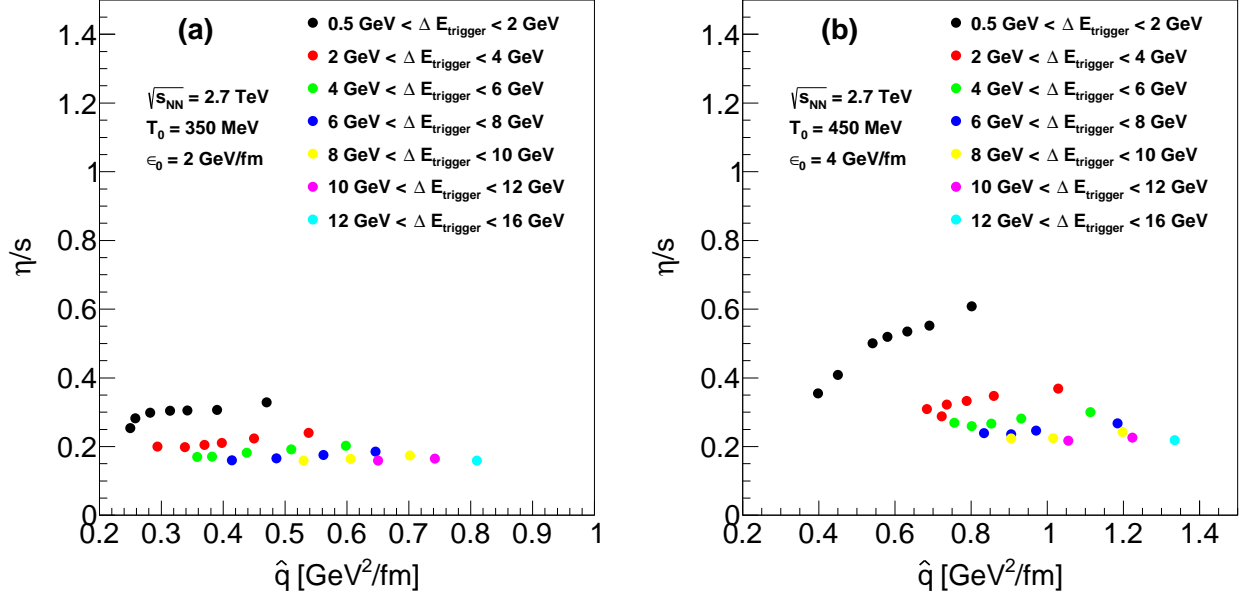


FIG. 7. Relation between η/s and \hat{q} for the away-side particle, for different trigger particle energy losses. The relation is obtained by plotting the value of these parameters that correspond to a given value of ΔE . The left (right) panel corresponds to a QGP temperature $T_0 = 350$ ($T_0 = 450$) MeV and a corresponding $\epsilon_0 = 2$ ($\epsilon_0 = 4$) GeV/fm in the $1 - d$ energy loss model.

momentum (squared) given from the partons to the medium and partons that travelled more medium (small to intermediate r) lose more energy, then, for bins with those values of r there should be a maximum in \hat{q} since on average the majority of partons are produced with that

value of momentum.

We can also study the behaviour of η/s either as a function of the energy lost or as a function of the amount of effective medium travelled by the partons. Figure 4 shows η/s as a function of ΔE of the away-side par-

ticle for the cases where the trigger particle loses the indicated amount of energy. Notice that as the trigger particle loses more energy, the away side particle energy loss starts at higher values, as expected. η/s is larger for the case where the trigger particle loses less energy. For all events but the ones where $0.5 < \Delta E_{\text{trigger}} < 2$ GeV the η/s values cluster around 0.2 (0.3) for the left (right) panel. In this case we have checked the behaviour of ΔE is dominated by events where the trajectory is tangential to the medium's surface, where, though L is small, the amount of traversed medium is also small and so is the average number of scatterings. In all cases η/s reaches limiting values which depend on the amount of energy lost by the trigger particle. Though the spread in values is less marked than for the case of \hat{q} , there is still a mild dependence on the energy loss by the trigger particle.

Figures 5 and 6 show η/s as a function of r for the trigger and associate particles, respectively. Notice that the value of this transport coefficient is more or less constant as a function of the location of the scattering center, where the fast parton is created. This can be understood from recalling that from Eq. (34) η/s is the ratio of the medium's travelled length to the average number of scatterings, scaled by the medium's temperature. Since the average number of scatterings is proportional to the travelled length, η/s is constant modulo the effect of the varying density as the medium expands. Also, η/s decreases (increases) with r for trigger (associate) particles. This can be understood as an effect caused by the amount of medium traversed by the corresponding particle. A trigger (associate) particle emitted close to the surface travels on average less (more) medium and this gets reflected in the behaviour of η/s with r .

Figure 7 shows the relation between η/s and \hat{q} for the away-side particle, for different trigger particle energy losses. This relation can be obtained from Eqs. (33) and (34) since for a given ΔE bin we have both a value of η/s and of \hat{q} . As the trigger particle loses more energy, the away side particle energy loss starts at higher values. Also, for all events but the ones where $0.5 < \Delta E_{\text{trigger}} < 2$ GeV the η/s values cluster around 0.2 (0.3) for the left (right) panel as \hat{q} varies. Once again, the behaviour of ΔE in this mentioned case is dominated by events where the trajectory is tangential to the medium's surface, where both L and $\langle n \rangle$ are small. We see that, though the dependence of η/s on \hat{q} is non-trivial due to effects caused by the ex-

panding medium, different \hat{q} values are described by more or less the same value of η/s . Overall, the milder dependence of η/s on events with different ΔE or coming from different locations r makes this transport coefficient to be a more accurate quantity to characterize the expanding plasma than \hat{q} .

IV. SUMMARY AND CONCLUSIONS

In conclusion we have shown that a non-trivial behaviour of the transport coefficients \hat{q} and η/s with the location of the hard scattering and with the energy lost characterizing the events is obtained for an expanding medium. To obtain this behaviour we resorted to model the amount of energy and momentum given to the medium by a fast moving parton in terms of linear viscous hydrodynamics and the amount of particles produced by this energy-momentum in terms of the Cooper-Frye formula. This procedure allows to obtain a probability distribution to compute the transport coefficients. To include the effect of an expanding medium we resorted to the model advocated in Ref. [18] tuned to describe LHC data [19]. The fast moving partons are produced in $2 \rightarrow 2$ parton events in p + p collisions with $\sqrt{s_{NN}} = 2.7$ TeV. This allows to characterize events where particles lose a given amount of energy or are produced at a given location within the medium.

The study shows that the expanding medium cannot be characterized by single values of \hat{q} or η/s , though the second one of these coefficients shows a milder dependence on r or ΔE . These results show that for conditions present in nuclear collisions at high energies, it is important to characterize the events in terms of a given observable, such as the amount of energy loss (missing p_t), before extracting a particular value for the transport coefficients.

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